The Accrual Anomaly in the Game-Theoretic Setting

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Summer 2010

Abstract

This paper proposes an alternative analysis of the accrual anomaly. It checks whether the accrual anomaly documented by Sloan (1996) exists in the game-theoretic setting. The game-theoretic tests identify the accrual anomaly in the following sense: a trading strategy that is based on the accrual component of earnings generates significant returns when one takes long positions in the market and invests more in companies with low accruals and less in companies with high accruals.

The game-theoretic approach has two main advantages: it does not assume any model for testing hypotheses, and it escapes from the EMH's joint hypothesis problem by identifying tests with trading strategies. The game-theoretic efficient market hypothesis states that a market is efficient when no trading strategy will multiply the capital it risks by a large factor. This prediction does not depend on probabilities and investors' preferences; instead it relies on the game-theoretic framework introduced by Shafer and Vovk in 2001. As Shafer and Vovk show, statistical testing can be alternatively understood in game-theoretic terms, because an event with very small probability happening is equivalent to a trading strategy multiplying the capital it risks by a large factor. Thus, we reject market efficiency at significance level α if the capital risked is multiplied by $1/\alpha$ or more.

1 Introduction

The relation between a company's earnings and stock returns has been widely analyzed. Following Ball and Brown (1968), many researchers have reported a positive association between a company's earnings and stock returns, which is generally attributed to the earnings' ability to summarize value relevant information (Sloan 1996). However, a number of studies prove that investors do not use correctly all available information in forecasting company's future performance (Bernard and Thomas (1990); Hand (1990); Maines and Hand (1996)). This evidence raises the possibility that the well documented relation between earnings and stock returns may, in fact, reflect investors' naive fixation on reported earnings rather than the earnings' ability to summarize value relevant information (Sloan 1996).

The Efficient Market Hypothesis (EMH) states that markets are "informationally efficient", i.e., security prices fully reflect all available information. Under this hypothesis, stocks always trade at their fair value, making it impossible for investors to purchase undervalued stocks or sell overvalued stocks. As such, it should be impossible to outperform the overall market through expert stock selection or market timing, and the only way an investor can possibly earn higher profits is by purchasing riskier securities.

Stating in 1970, Fama first emphasized that the EMH suffers from a joint hypothesis problem. He underlined that market efficiency per se is not testable, it must be tested jointly with some model of equilibrium, an asset-pricing model (Fama 1970). In other words, we can only check whether information is properly reflected in prices in the context of a pricing model that defines the meaning of "properly". That is why, in order to test the EMH, one should develop a model that specifies probabilities and utilities for a marginal investor. In fact, because of the joint hypothesis problem, precise inferences about the degree of market efficiency are likely to remain impossible (Fama 1970).

As documented by Sloan (1996), the EMH is violated due to "the accrual anomaly". Investors who focus on net income rather than net operating cash flows tend to overestimate the predictive ability of accruals and underestimate the predictive ability of cash flows. Sloan's study shows that a trading strategy that takes a long position in companies with low accruals and a short position in companies with high accruals generates significant abnormal returns.

In this paper, we want to study the accrual anomaly using the game-theoretic framework which was first introduced by Glenn Shafer and Vladimir Vovk in *Probability and Finance: It's only a Game!* (2001). This approach is beneficial for several reasons. Particularly, it does not assume any model for testing hypotheses, and it escapes from the EMH's joint hypothesis problem by identifying tests with trading strategies. Game-theoretic tests with trading strategies make it possible to identify anomalies and measure the degree of market inefficiency related to a particular anomaly without modeling probabilities and utilities.

The *Game-Theoretic Efficient Market Hypothesis* (GEMH) states that no trading strategy will multiply the capital it risks by a large factor. This hypothesis can be tested directly with no further assumptions about probabilities or investors' preferences. In order to test the GEMH, we develop a trading strategy that depends on the accrual component of earnings and takes a long position in the market. In particular, the amount of wealth invested in each company depends on accruals: more money is invested in companies with low accruals and less with high accruals.

Our game-theoretic analysis, which does not assume any model for testing hypotheses, but depends on the developed trading strategy, reveals a strong evidence that one can earn significant returns by taking a long position in the market during 2002-2008 and investing more in companies with low accruals and less with high accruals. Although we have not considered a short position for a trading strategy, our results substantiate that the accrual anomaly continues to persist on recent data.

The reminder of the paper is organized as follows. Section 2 explains the accrual anomaly. Section 3 presents the game-theoretic framework. Section 4 develops a trading strategy that depends on the accrual component of reported earnings. Section 5 describes sample formation and variable measurement. Section 6 reports the results. Section 7 talks about future research, and section 8 concludes the paper.

2 The accrual anomaly

A company's earnings are comprised of two components: accruals and cash flows. The accrual component includes accruals, deferrals, allocations and valuations, and the cash flows component consists of net cash received or paid within a period. Many parts of the accrual component are discretionary, i.e., subject to management's judgments, while the cash flows component is much less subjective, i.e., less subject to distortion.

In 1996, Sloan documented that the cash flow component of earnings is more closely associated with future income and stock returns than is the accrual component of earnings. He referred this effect to the fact that accruals reverse faster than earnings in subsequent periods and are less persistent than cash flows. Accruals tend to be less persistent than net operating cash flows because of intentional or unintentional managerial errors in forecasting future cash flows. In particular, managers may use positive accruals to elevate earnings when operating cash flows are low and use negative accruals to reduce earnings when operating cash flows are high.

Therefore, an understanding of what accruals are and why they are less persistent than net operating cash flows is important for analysts and investors. If sufficiently large numbers of investors focus on total income and pass over the information in net operating cash flows and accruals, prices may not properly reflect the company's economic position. The result is mispriced securities and "the accrual anomaly" (Livnat and Santicchia 2006).

3 The game-theoretic framework

Consider a perfect-information L-period game between two players, Speculator and Market. Denote by N the number of companies in the market. Let the capital risked by Speculator's strategy be his initial capital, $\mathcal{K}_0$, in the game. For simplicity, assume that $\mathcal{K}_0$ is equal to 1. At the beginning of each period, knowing the accrual component of earnings from the previous period, a_{ij-1}, Speculator decides on a fraction of his current wealth, λ_{ij-1}, to invest in company i. At the end of each period Market chooses simple returns r_{ij}, $i = 1...N, j = 1...L$. Let $\lambda_j = (\lambda_{1j}, \lambda_{2j}, ..., \lambda_{Nj})$, $r_j = (r_{1j}, r_{2j}, ..., r_{Nj})$, and $e = (1, 1, ..., 1)$.

The following protocol explains how Speculator's capital process, $\mathcal{K}_0, \mathcal{K}_1, ..., \mathcal{K}_L$, is determined:

Protocol 1

$\mathcal{K}_0 = 1.$
FOR $j = 1, 2, ...L$:
Speculator announces $\lambda_{j-1} \in [0,1]^N, \lambda_{j-1}e^T = 1.$
Market announces $r_j \in (-1, \infty)^N.$
$\mathcal{K}_j := \mathcal{K}_{j-1}\left(1 + \lambda_{j-1}r_j^T\right).$

The only risk considered in this Protocol is the risk of bankruptcy. The natural question may arise now: what does it mean for the strategy for Speculator to risk only the initial capital $\mathcal{K}_0$ in the game? It means that the move λ_0 specified by the strategy risks no more than $\mathcal{K}_0$, the move λ_1 risks no more than $\mathcal{K}_1$, and so on. This Speculator's strategy guaranties that $\mathcal{K}_j$ will be nonnegative for all j, $j = 1...L$, no matter how Market moves.

The requirement that Speculator risks only his current capital $\mathcal{K}_{j-1}$ on the jth round prevents him from going short. In this case Market can bankrupt him by choosing such r_j that $\lambda_{j-1} r_j^T = -1$.

As was mentioned above, two players, Speculator and Market, move in sequence and see each other's moves. Within minimal constraints, each player can move as he pleases. Modeling the market as a single player who can play strategically against Speculator may look suspicious, but it is appropriate since our mathematical framework guarantees that Speculator's capital will be nonnegative, no matter how Market moves. It is clear that the case where Market plays strategically is the worst for Speculator's trading strategy.

The game-theoretic approach has the traditional intuition for market efficiency: opportunities for easy money have already been eliminated. The GEMH states that there is no trading strategy that multiplies the capital it risks by a large factor. This idea of measuring the profitability of trading strategy by the factor by which it multiplies the capital it risks is new. It was derived from the game-theoretic analysis of classical probability theory done by Shafer and Vovk in 2001.

In classical probability theory it is common to reject a hypothesis when a preselected event to which the hypothesis assigns small probability (say α, $\alpha < 5\%$) happens. As Shafer and Vovk show, statistical testing can be alternatively understood in game-theoretic terms, because an event with very small probability happening is equivalent to a trading strategy multiplying the capital it risks by a large factor. This is equivalent to rejecting the hypothesis because the expected values it gives, when interpreted as prices, allow Speculator to multiply the capital he risks by $1/\alpha$ or more. For example, multiplying one's capital by 1000 is the same as an event of probability 0.001 happening. (Shafer and Vovk 2001, pp.194-197).

In order to test market efficiency, we select a significance level α (say $\alpha \leq 5\%$) and a trading strategy for Speculator that is based on the accrual component of earnings and risks only the initial capital.

- TESTING: The GEMH is rejected if Market moves in such a way that $\mathcal{K}_L > 1/\alpha$ when Speculator follows his trading strategy.

4 Developing the trading strategy

The literature on the EMH has paid a little attention to the capital risked by a trading strategy. EMH's studies usually develop self-financing strategies that sell stocks short, that is why, the capital they risk may be much larger than meets the eye.

In this study we consider a strategy that depends on a company's accruals and stakes only the initial capital. Speculator takes a long position in the market, invests more in "low accruals companies" and less in "high accruals companies", and risks only his current capital.

We would like to describe Speculator's trading strategy using some analytical function that has nice properties and decreases in the increasing rate. That is why, we make an assumption that a negative exponential function is the most proper for our analysis. The following description will clarify in more detail how the trading strategy has been developed.

Let r_{ij} denote the stock return of firm i for period j, a_{ij-1} represent the level of accruals of firm i in period *j-1*, N be the number of firms on the market, and L be the number of periods.

Let λ_{ij-1} denote the fraction of initial wealth, invested in company i in period j, then:

$$\sum_{i=1}^{N} \lambda_{ij-1} = 1, j = 1...L.$$

Suppose there is an unknown constant $\rho \in \mathbb{R}^{+}$, such that:

$$\lambda_{ij-1} = \frac{e^{-\rho a_{ij-1}}}{\sum_{i=1}^{N} e^{-\rho a_{ij-1}}}, i = 1...N, j = 1...L.$$

In order to find ρ, we solve an optimization problem using accruals for period *j-1* and stock returns for period j.

Let $\Lambda = [\lambda_{ij-1}], i = 1...N, j = 1...L$. Then, the profit maximization problem is:

$$\max_{\Lambda} \sum_{j=1}^{L} \sum_{i=1}^{N} \lambda_{ij-1}(1 + r_{ij}),$$

which is equivalent to:

$$\max_{\rho \in \mathbb{R}^{+}} \sum_{j=1}^{L} \sum_{i=1}^{N} \frac{e^{-\rho a_{ij-1}}}{\sum_{i=1}^{N} e^{-\rho a_{ij-1}}}(1 + r_{ij}),$$

For simplicity, we designate this objective function by f:

$$f(\rho) := \sum_{j=1}^{L} \sum_{i=1}^{N} \frac{e^{-\rho a_{ij-1}}}{\sum_{i=1}^{N} e^{-\rho a_{ij-1}}}(1 + r_{ij}).$$

5 Sample and Measurement

In order to test the GEMH, we use all the firms with available data in the CRSP/COMPUSTAT Merged (Fundamentals Annual). The study employs financial statement data for 18 years starting in 1990 and ending in 2008. We select all the firms that were active every year during 1990-2001 and 2002-2008. Firms with missing data are deleted from the sample. The final sample consists of 1,845 firms that had been active every year during 1990-2001 and 2,930 firms that had been active every year during 2002-2008.

We consider data from the first period, 1990-2001, as the training set and use it to solve the optimization problem. Data from the second period, 2002-2008, is used as the testing set where we check how profitable Speculator's trading strategy is.

The financial variables of interest are accruals and stock returns. Accruals are calculated as the difference between net income and cash flow from operations, standardized by average total assets:

$$a_{ij-1} = (NI_{ij-1} - CFO_{ij-1})/ATA_{ij-1},$$

where

a_{ij-1} is the accrual component of earnings of company i at the end of period *j-1*;

NI_{ij-1} is the reported net income of company i at the end of period *j-1*;

$ATA_{ij-1} = \frac{TA_{ij-1}+TA_{ij-2}}{2}$ are the average total assets of company i at the end of period *j-1*.

Stock returns measure the percentage change in value over a period of time, and are calculated as:

$$r_{ij} = \frac{p_{ij}f_{ij} + d_{ij}}{p_{ij-1}} - 1,$$

where

r_{ij} is the stock return of company i at the end of the period j;

p_{ij} is the closing price of company i at the end of the period j;

$f_{ij} = \frac{\text{number of shares outstanding in period } j}{\text{number of shares outstanding in period } j\text{-}1}$ is the split factor;

d_{ij} is the per share value of dividends of company i at the end of period j.

6 Results

This section presents solutions of the optimization problem and results of the game-theoretic testing. We solve the optimization problem, which was stated in §3, using the programming language C++. For any given optimization problem, it is a good idea to compare several of the available algorithms that are applicable to that problem in general. In order to find an optimal constant ρ, we employ different NLopt Algorithms.

We consider the following algorithms in our study:

- NLOPT_GN_DIRECT, a dividing rectangles algorithm for nonlinear global optimization, described in [6].

- NLOPT_GN_ISRES, an algorithm for nonlinear global optimization based on the method described in [9].

- NLOPT_GN_ORIG_DIRECT, algorithm for nonlinear global optimization based on the method described in [4].

Table 1 reports outcomes that were found by three algorithms on the training set. We can see that all tree algorithms generated about the same results. In particular, the optimal constant ρ^* is approximately 2.37, and the value of the objective function in 2.37 is approximately 27.96. These results indicate that Speculator who takes a long position in the market, uses the trading strategy that depends on companies' accruals, and invests \$1 every year will earn almost \$28. Nevertheless, it is too soon to make any conclusions about the profitability of this trading strategy. We need to check whether the optimal constant ρ^* works on the testing set.

Table 2 shows the values of the objective function $f(\rho)$ on the testing set. It can be seen that $\rho^* \approx 2.37$ still works on the testing set. Speculator who takes a long position during 2002-2008, uses accruals for his trading strategy, and invests \$1 every year will make a profit of $\approx$ \$22.5. Thus, using different NLopt Algorithms, we found the best optimal ρ^* (≈ 2.37) for Speculator's trading strategy. For simplicity, we consider $\rho^* = 2.3825$ in the further game-theoretic testing.

From the assumptions made above we know that the amount invested in each company every year depends on the respective number of accruals and the constant $\rho^* = 2.3825$, that is:

$$\lambda_{j-1} = (\frac{e^{-2.3825 a_{1j-1}}}{\sum_{i=1}^{N} e^{-2.3825 a_{ij-1}}}, ..., \frac{e^{-2.3825 a_{Nj-1}}}{\sum_{i=1}^{N} e^{-2.3825 a_{ij-1}}}).$$

We can now state precisely two protocols for the game involving Speculator and Market:

PROTOCOL 2.1 (*when Speculator risks all his initial capital*)

$\mathcal{K}_0 = 1.$
FOR $j = 1, 2, ...L$:
  Speculator announces $\lambda_{j-1} \in [0,1]^N, \lambda_{j-1}e^T = 1.$
  Market announces $r_j \in (-1, \infty)^N.$
  $\mathcal{K}_j := \mathcal{K}_{j-1}\left(1 + \lambda_{j-1}r_j^T\right).$

PROTOCOL 2.2 (*when Speculator risks a half of his initial capital*)

$\mathcal{K}_0 = 1.$
FOR $j = 1, 2, ...L$:
  Speculator announces $\lambda_{j-1} \in [0,1]^N, \lambda_{j-1}e^T = 1.$
  Market announces $r_j \in (-1, \infty)^N.$
  $\mathcal{K}_j := \mathcal{K}_{j-1}\left(1 + \frac{\lambda_{j-1}r_j^T}{2}\right).$

Table 3 shows the results of the game-theoretic testing. In order to find Speculator's capital process, we use Protocol 2.1 and 2.2. If Speculator follows his trading strategy and invests all his initial capital at the beginning of each year, he will make a total profit of \$263.068 at the end of the last year. If Speculator employs his trading strategy and invests only a half of the initial capital at the beginning of each year, he will earn profit of \$47.452 at the end of the last year.

According to the GEMH, there is no trading strategy that will multiply the capital it risks by a large factor. Recall that the GEMH is rejected if Market moves in such a way that $\mathcal{K}_L > 1/\alpha$ when Speculator follows his trading strategy. In our case we can observe that the developed trading strategy for Speculator has produced $\mathcal{K}_L > 1/\alpha$. In Protocol 2.1 Speculator's final capital $\mathcal{K}_L$ is equal to \$263.068, and the inequality $\mathcal{K}_L > 1/\alpha$ holds for all $\alpha > 0.0038$. In Protocol 2.2 Speculator's final profit $\mathcal{K}_L$ is \$47.452, and the inequality $\mathcal{K}_L > 1/\alpha$ holds for all $\alpha > 0.02107$. Therefore, our test rejects the hypothesis of market efficiency when α is sufficiently small (say 1% in Protocol 2.1 and 5% in Protocol 2.2).

It is interesting to see how our game-theoretic results will change when we allow ρ in Protocol 1 to be negative. This change will permit Speculator to use the opposite trading strategy: now he will invest more in companies with high accruals and less in companies with low accruals. In oder to see how this modification will influence our final results, we solve the optimization problem stated in §3. Now the upper bound of ρ is set to be 0, that is $\rho \in \mathbb{R}^-$.

Solving the optimization problem, we find that the optimal constant ρ^* is approximately -49.89, and the maximum of the objective function on the testing set is \$8.99. Thus, Speculator, who invests \$1 every year, earns profit of almost \$9 at the end of the last year. Let us verify how this trading strategy works in the game-theoretic protocol.

When we set $\rho = -49.89$ in Protocol 1, Speculator's final capital becomes insignificant (even if $\alpha \leq 10\%$). Table 4 shows the results of the game-theoretic testing. The trading strategy that stakes \$1 at the beginning of 2002, risks all the initial capital in every period, and invests more in "high accruals companies" makes profit of \$6.422 at the end of 2008. Similar trading strategy that risks only a half of the initial capital generates profit of \$3.079 at the end of 2008. Thus, we cannot reject the GEMH even if we set a significance level α to be 10%.

So far we have shown that Speculator's trading strategy yields significant positive returns when he takes a long position in the market, risks only his initial capital, and invests more in companies with low accruals and less with high accruals. However, we have not considered a case when Speculator can take a long as well as a short position in the market. Also, we have not taken into account the issue of transaction costs in our game-theoretic testing. Therefore, the finding that Speculator's trading strategy multiplies the capital it risks by a large factor does not necessary imply the existence of unexploited profit opportunities. Section 6 covers some ideas how to incorporate transaction costs into the game-theoretic testing, but we leave this part for the future research.

Finally, we would like to know whether the accrual anomaly documented by R. Sloan (1996) exists on our data set. According to Sloan, a trading strategy that takes a long position in the stock of firms that report low levels of accruals and a short position in the stock of firms that report high levels of accruals generates positive abnormal returns. In order to examine returns of this trading strategy, we partially replicate Sloan's study. First, we rank companies based on the amount of the accrual component of earnings and assign them in equal numbers to ten portfolios. Second, we form a hedge portfolio by taking a long position in the lowest accrual portfolio and an equally valued short position in the highest accrual portfolio. Then we calculate the return to the hedge portfolio by subtracting the return to highest accrual portfolio from the return to the lowest accrual portfolio. The second and third columns of table 5 report the returns to the lowest and highest accrual portfolio respectively. It can be seen that the return to the hedge portfolio is positive in four of the five years examined, it ranges from 7.64% for 2003 to -5.78% for 2007. Thus, we can make a rough conclusion from this short analysis: the accrual anomaly documented by Sloan (1996) still persists on our data set.

Table 1: Optimization problem: solutions

Time period	ρ^*	$f(\rho^*)$	**Algorithm**
1990-2001	2.3825	27.9654	GN_DIRECT
1990-2001	2.3668	27.9653	GN_ISRES
1990-2001	2.3663	27.9653	GN_ORIG_DIRECT

Table 2: Validation of ρ^*

Time period	ρ^*	$f(\rho^*)$	**Algorithm**
2002-2008	2.3825	22.5143	GN_DIRECT
2002-2008	2.3668	22.4745	GN_ISRES
2002-2008	2.3663	22.4732	GN_ORIG_DIRECT

7 Future research

In the previous section we have identified the accrual anomaly and found that the trading strategy, which was developed in section 3, multiplies the capital it risks by the large factor, but we have not mentioned anything about transaction costs and alternative trading strategies. Therefore, the finding that Speculator's trading strategy that depends on the accrual component of earnings multiplies the capital it risks by a large factor does not necessary imply the existence of unexploited profit opportunities. The transaction costs associated with implementing this strategy in real time are non-trivial. In this

Table 3: Speculator's capital process when $\rho > 0$

	Year	**Value of ρ^***	**Capital(\$, $\mathcal{K}_j$)**	**Value of $\hat{\alpha}$**
Strategy that risks all the initial capital	2002	2.3825	1	
	2003	2.3825	4.2188	0.23703
	2004	2.3825	11.0853	0.09021
	2005	2.3825	126.369	0.00791
	2006	2.3825	247.155	0.00405
	2007	2.3825	414.765	0.00241
	2008	2.3825	263.068	0.0038
Strategy that risks a half of the initial capital	2002	2.3825	1	
	2003	2.3825	2.6094	0.38323
	2004	2.3825	4.7329	0.21101
	2005	2.3825	29.3435	0.03408
	2006	2.3825	43.367	0.02305
	2007	2.3825	58.0717	0.01722
	2008	2.3825	47.4521	0.02107

section, we give an idea how the game-theoretic approach can incorporate the issue of transaction costs into its testing.

From an economic perspective it is not clear how the significance of Speculator's returns measures inefficiency. The game-theoretic approach, however, proposes a natural way to measure the relative inefficiency of the market. The degree of market inefficiency implied by the accrual anomaly (or any other anomaly) is simply the level of transaction costs needed to eliminate the strategy's profit and hence the test's significance.

Consider the effect of transaction costs on Speculator's capital in Protocol 1. At the beginning of period j Speculator decides to invest $\mathcal{K}_{j-1}\lambda_{ij-1}$ in the company i. But he has already invested $\mathcal{K}_{j-2}\lambda_{ij-2}(1 + r_{ij-1})$ in the company i. So the amount of capital he shifts at the beginning of period j is $|\mathcal{K}_{j-1}\lambda_{ij-1} - \mathcal{K}_{j-2}\lambda_{ij-2}(1 + r_{ij-1})|$. Assume that the cost of this shift is a fraction τ of the amount shifted, then Speculator's capital at the end of period

Table 4: Speculator's capital process when $\rho < 0$

	Year	Value of ρ^*	Capital($, $\mathcal{K}_j$)	Value of $\hat{\alpha}$
	2002	-49.89	1	
	2003	-49.89	3.017	0.331
Strategy that risks all the initial capital	2004	-49.89	4.412	0.227
	2005	-49.89	6.99	0.143
	2006	-49.89	5.786	0.173
	2007	-49.89	6.287	0.159
	2008	-49.89	6.422	0.156
	2002	-49.89	1	
	2003	-49.89	2.008	0.498
Strategy that risks a half of the initial capital	2004	-49.89	2.473	0.404
	2005	-49.89	3.195	0.313
	2006	-49.89	2.92	0.342
	2007	-49.89	3.047	0.328
	2008	-49.89	3.079	0.325

j is:

$$\mathcal{K}_j := \mathcal{K}_{j-1}\left(1+\sum_{i=1}^{N}\lambda_{ij-1}r_{ij}\right) - \sum_{i=1}^{N}\left|\mathcal{K}_{j-1}\lambda_{ij-1} - \mathcal{K}_{j-2}\lambda_{ij-2}(1+r_{ij-1})\right|\tau.$$

The significance of Speculator's returns can be measured by the value of τ that reduces a final capital, $\mathcal{K}_L$, to 20 or even to 1. Let τ_{20} denote the value of τ that reduces $\mathcal{K}_L$ to 20, and τ_1 the value that reduces it to 1. Then we say that τ_1 is the economic significance of the game-theoretic results.

As far as alternative trading strategies is concerned, we plan to develop a strategy that takes a long as well as a short position in the market and is based not only on the accrual component of earnings but on some other economic factors. Moreover, we want to modify our sample by allowing the number of firms to vary across time.

Table 5: Returns to the hedge portfolio taking a long position in the lowest accrual portfolio and a short position in the highest accrual portfolio

Year	Returns to the lowest accrual portfolio	Returns to the highest accrual portfolio	Returns to the hedge portfolio
2003	0.2686	0.1922	0.0764
2004	0.0959	0.0072	0.0887
2005	0.0617	0.0502	0.115
2006	-0.0097	-0.0096	0.0086
2007	-0.0493	0.0086	-0.0578

8 Conclusion

In this study we have shown an alternative approach to a validation of the accrual anomaly. In particular, we have employed the game-theoretic framework introduced by Shafer and Vovk in *Probability and Finance: It's only a Game!* (2001). This approach does not assume any model for testing hypotheses, and it escapes from the EMH's joint hypothesis problem by identifying tests with trading strategies. In this study we have stated the game-theoretic efficient market hypothesis(GEMH) that there is no trading strategy that multiplies the capital it risks by a large factor.

In order to prove the existence of the accrual anomaly and to test the GEMH, we have developed a trading strategy for Speculator and showed that it yields significant returns. This strategy takes a long position in a market, depends on the accrual component of companies' earnings, and invests more in "low accruals companies" and less in "high accruals companies". Although we have not considered a short position for a trading strategy, our results are consistent with previous research.

Our results raise additional issues for future research. Of particular interest is a trading strategy for Speculator that depends on the accrual component of earnings and takes a long as well as a short position in the market. Also, we would like to examine the issue of transaction costs since our current findings do not necessary imply the existence of unexploited profit opportunities.

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